In class exercise problem

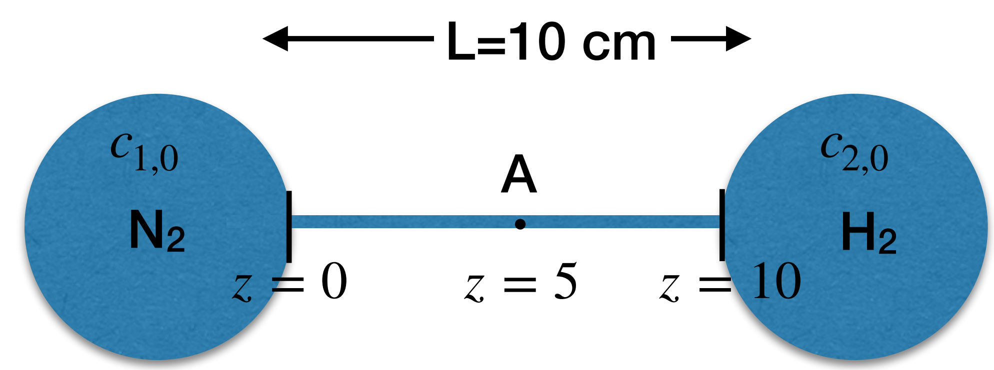
In the diffusion apparatus shown here, a large, well-mixed and infinitely large N_2 reservoir is separated from an infinitely large, well-mixed H_2 reservoir using a small tube with length of 10 cm. D for H_2 is 0.1 cm^2/s and for N_2 is 0.1 cm^2/s. Calculate mass, molar and volume average velocities at point A at the steady state. Also calculate the flux of N_2 at point A.

N_2 is species 1, H_2 is species 2

$$v = \text{average velocity of moles} \ = \ 0$$

$$v^v = \text{average velocity of volumes} \ = \ 0$$

$$v^m = \text{average velocity of mass} \ \neq \ 0 \ = \ \mathrm{w}_1 \mathrm{v}_1 + \mathrm{w}_2 \mathrm{v}_2$$



$$\mathbf{c_{1,0} = c_{2,0}}$$

Using average velocity of moles is convenient because it is zero

$$n_1 \ = \ c_1 v_1 \ = \ j_1 + c_1 v \qquad j_1 \ = -D_1 c \nabla y_1 \qquad v = 0$$

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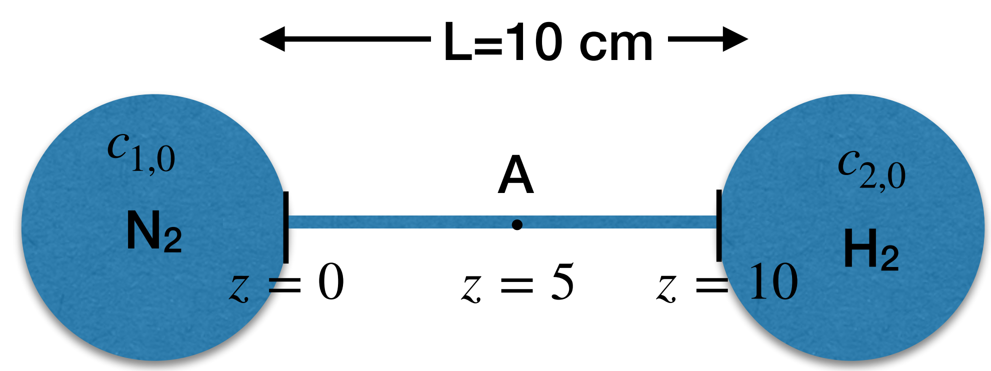
Because the transport is purely diffusive in our reference, we can use the thin film results

$$j_{N_2} = j_1 = -D_1 \frac{dc_1}{dz} = D_1 \frac{(c_{1,0} - c_{1,L})}{L} \ = \ D_1 \frac{c_{1,0}}{L}$$

$$c_{1,A} = c_{1,0} + (c_{1,L} - c_{1,0}) \frac{z_A}{L} = c_{1,0}\left(1 - \frac{5}{10}\right) = \frac{c_{1,0}}{2}$$

$$c_{1,A} = \frac{c_{1,0}}{2} \qquad j_1 = D_1 \frac{c_{1,0}}{L}$$



$$\mathbf{c_{1,0} = c_{2,0}}$$

$$c_{1,A}\, v_{1,A} = j_{1,A}$$

$$\Rightarrow v_{1,A} = \frac{j_{1,A}}{c_{1,A}}$$

$$= 2D_1 \frac{c_{1,0}}{L c_{1,0}} = \frac{2D_1}{L} = \frac{2 * 0.1 * 10^{-4}}{0.1} = 2 * 10^{-4}\ ms^{-1}$$

$\rightarrow$

N_2 moves from left to right

Similarly,

$$v_{2,A} = \frac{j_{2,A}}{c_{2,A}} = -\frac{2D_2}{L} = -2 * 10^{-4}\ ms^{-1}$$

H_2 moves from right to left $\leftarrow$

$$v^m = \text{average velocity of mass} = \mathrm{w}_1 \mathrm{v}_1 + \mathrm{w}_2 \mathrm{v}_2$$

$$w_{1,A} = \frac{c_{1,A} * MW_1}{c_{1,A} * MW_1 + c_{2,A} * MW_2} = \frac{MW_1}{MW_1 + MW_2} = \frac{28}{28+2} = 28/30 \qquad w_2 = 2/30$$

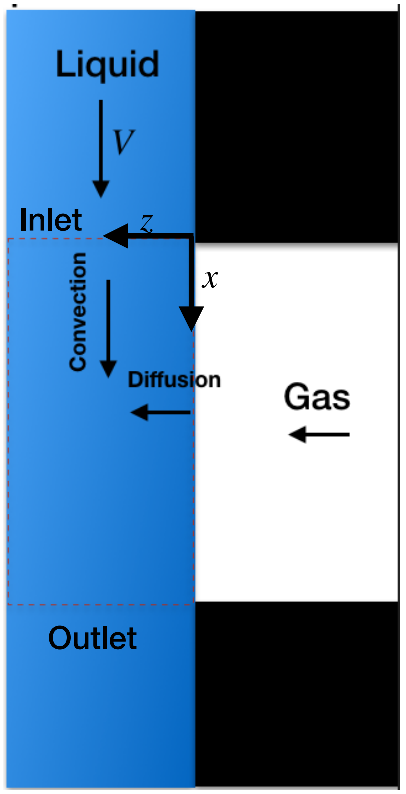
$$\mathbf{v^m \text{ at } A = w_{1,A} v_{1,A} + w_{2,A} v_{2,A} = (28/30 * 2 - 2/30 * 2) * 10^{-4}\ ms^{-1}} = 2 * 26/30 * 10^{-4}\ ms^{-1}$$

Exercise problem 1: Carbon capture by falling amine film

A promising technique to remove carbon dioxide is to trap it in a falling amine film by absorption. Consider a 1 m tall falling pure amine film contacted with CO_2 at the surface, immediately reaching saturation at the surface (z = 0). Calculate the minimum diffusion coefficient, D, needed to achieve a CO_2 concentration that is at least 50% of its saturation value at the outlet (x = 1 m) of the stream (bottom) at z (depth inside film) = 1 cm.

The film is falling with a velocity of 1 cm/s.

x and z are specified; x = 1 m, z = 1 cm (outlet)



$$\frac{c(z,x) - c_S}{c_\infty - c_S} = erf\,\zeta \qquad \Rightarrow erf\,\zeta = 0.5$$

$$\zeta = \frac{z}{\sqrt{4D\frac{x}{V}}} = 0.48$$

$$\frac{0.01}{0.48} = \sqrt{4D\frac{1}{0.01}} \qquad D = 1.1 * 10^{-6}\ m^2\, s^{-1}$$

Exercise problem 2

In the diffusion apparatus shown here, an infinitely large, well-mixed toluene reservoir is separated from an infinitely large, well-mixed benzene reservoir using a small tube with length of 10 cm. D is same for both liquids and is 10^{-4} cm^2/s. Calculate mass and molar average velocities at points B at the steady state. Assume volume average velocity to be zero.

Toluene is species 1, Benzene is species 2

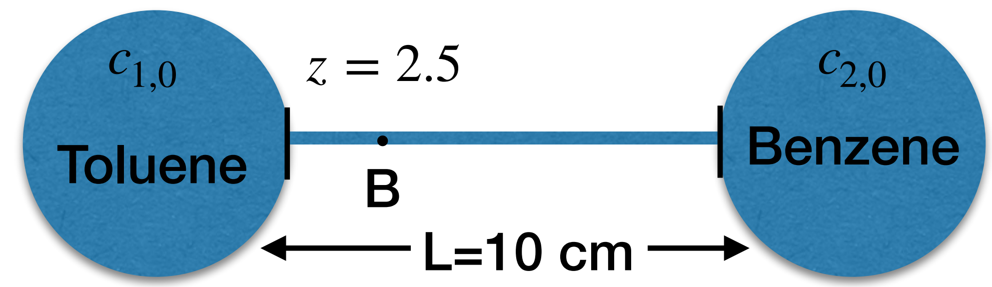
Lets assume

$$v = \text{average velocity of moles} \neq 0 = y_1 v_1 + y_2 v_2$$

$$v^v = \text{average velocity of volumes} = 0$$

Lets assume

$$v^m = \text{average velocity of mass} \neq 0 = w_1 v_1 + w_2 v_2$$



$$\mathbf{c_{1,0} = c_{2,0}}$$

Using average velocity of volumes is convenient because it ~ zero (similar molar volume or density)

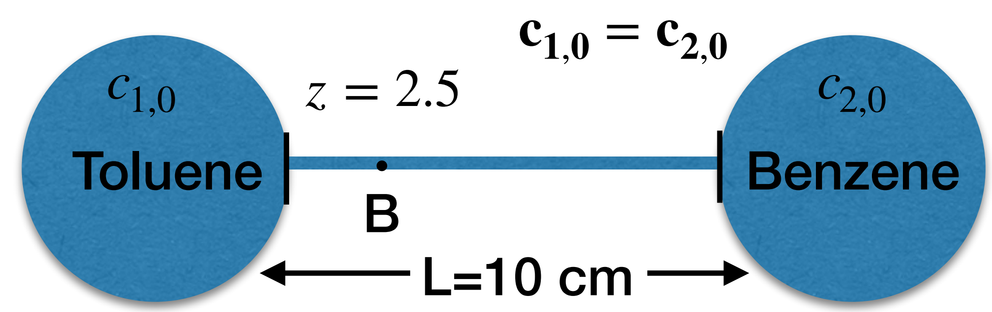
$$n_1 = c_1 v_1 = j_1^v + c_1 v^v \qquad j_1^v = c_1(v_1 - v^v) = -D\nabla c_1$$

$$n_1 = c_1 v_1 = j_1 = -D\nabla c_1$$

Because the transport is purely diffusive in our reference, we can use thin film results

$$j_{toluene} = j_1 = -D\frac{dc_1}{dz} = D\frac{(c_{1,0} - c_{1,L})}{L} = D\frac{c_{1,0}}{L}$$

$$c_{1,B} = c_{1,0} + (c_{1,L} - c_{1,0})\frac{z_B}{L} = c_{1,0}\left(1 - \frac{2.5}{10}\right) = \frac{3}{4}c_{1,0}$$



$$n_1 = c_1 v_1 = j_1 = -D \nabla c_1$$

$$j_{toluene} = j_1 = D\frac{c_{1,0}}{L}$$

$$c_{1,B} = \frac{3}{4} c_{1,0}$$

$$c_{1,B}\, v_{1,B} = j_{1,B} \qquad v_{1,B} = \frac{j_{1,B}}{c_{1,B}} = \frac{4}{3} D \frac{c_{1,0}}{L c_{1,0}} = \frac{4D}{3L} = \frac{4 * 1 * 10^{-8}}{3 * 0.1} = 1.33 * 10^{-7}\, ms^{-1} \rightarrow$$

toluene moves from left to right

Similarly, $$v_{2,B} = \frac{j_{2,B}}{c_{2,B}} = -\frac{4D}{L} = -\frac{4 * 1 * 10^{-8}}{0.1} = -4 * 10^{-7}\, ms^{-1}$$

Benzene moves from right to left $\leftarrow$

$$c_{2,B} = \frac{1}{4} c_{2,0}$$

$$v^m = \text{average velocity of mass} = \mathrm{w}_1 \mathrm{v}_1 + \mathrm{w}_2 \mathrm{v}_2$$

$$w_{1,B} = \frac{c_{1,B} * MW_1}{c_{1,B} * MW_1 + c_{2,B} * MW_2} = \frac{3MW_1}{3MW_1 + MW_2} = \frac{3 * 92.1}{3 * 92.1 + 78.1} = 0.78$$

$$w_{2,B} = \frac{3MW_2}{3MW_1 + MW_2} = 0.22 \qquad v^m = w_1 v_1 + w_2 v_2 = \frac{(4D/L)}{3MW_1 + MW_2}\left[MW_1 - MW_2\right] = 1.57 * 10^{-8}\, ms^{-1}$$

$$y_1 = 3/4 \qquad y_2 = 1/4 \qquad v = y_1 v_1 + y_2 v_2 = 0$$